

# The observation of long-range three-body Coulomb effects in the decay of $^{16}\text{Ne}$

K.W. Brown,<sup>1</sup> R.J. Charity,<sup>1</sup> L.G. Sobotka,<sup>1</sup> Z. Chajecki,<sup>2</sup> L.V. Grigorenko,<sup>3,4</sup> I.A. Egorova,<sup>5</sup> Yu.L. Parfenova,<sup>6,7</sup> M.V. Zhukov,<sup>8</sup> S. Bedoor,<sup>9</sup> W.W. Buhro,<sup>2</sup> J.M. Elson,<sup>1</sup> W.G. Lynch,<sup>2</sup> J. Manfredi,<sup>2</sup> D.G. McNeel,<sup>9</sup> W. Reviol,<sup>1</sup> R. Shane,<sup>2</sup> R.H. Showalter,<sup>2</sup> M.B. Tsang,<sup>2</sup> J.R. Winkelbauer,<sup>2</sup> and A.H. Wuosmaa<sup>9</sup>

<sup>1</sup>*Departments of Chemistry and Physics, Washington University, St. Louis, Missouri 63130, USA.*

<sup>2</sup>*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

<sup>3</sup>*Flerov Laboratory of Nuclear Reactions, JINR, Dubna, RU-141980, Russia*

<sup>4</sup>*Russian Research Center “The Kurchatov Institute”, Kurchatov sq. 1, RU-123182 Moscow, Russia*

<sup>5</sup>*Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980, Russia*

<sup>6</sup>*Flerov Laboratory of Nuclear Reactions, JINR, Dubna, RU-141980 Russia*

<sup>7</sup>*Skobel’syn Institute of Nuclear Physics, Moscow State University, 119991 Moscow, Russia*

<sup>8</sup>*Fundamental Physics, Chalmers University of Technology, S-41296 Göteborg, Sweden*

<sup>9</sup>*Department of Physics, Western Michigan University, Kalamazoo, Michigan 49008, USA.*

The interaction of an  $E/A=57.6$ -MeV  $^{17}\text{Ne}$  beam with a Be target was used to populate levels in  $^{16}\text{Ne}$  following neutron knockout reactions. The decay of  $^{16}\text{Ne}$  states into the three-body  $^{14}\text{O}+p+p$  continuum was observed in the High Resolution Array (HiRA). For the first time for a  $2p$  emitter, correlations between the momenta of the three decay products were measured with sufficient resolution and statistics to allow for an unambiguous demonstration of their dependence on the long-range nature of the Coulomb interaction. Contrary to previous measurements, our measured limit  $\Gamma < 80$  keV for the intrinsic decay width of the ground state is not in contradiction with the small values (of the order of keV) predicted theoretically.

PACS numbers: 25.10.+s, 23.50.+z, 21.60.Gx, 27.20.+n

*Introduction* — Two-proton ( $2p$ ) radioactivity [1] is the most recently discovered type of radioactive decay. It is a facet of a broader three-body decay phenomenon actively investigated within the last decade [2]. In binary decay, the correlations between the momenta of the two decay products are entirely constrained by energy and momentum conservation. In contrast for three-body decay, the corresponding correlations are also sensitive to the internal nuclear structure of the decaying system and the decay dynamics providing, in principle, another way to constrain this information from experiment. In  $2p$  decay, as the separation between the decay products becomes greater than the range of the nuclear interaction, the subsequent modification of the initial correlations is determined solely by the Coulomb interaction between the decay products. As the range of the Coulomb force is infinite, its long-range contribution to the correlations can be substantial, especially, in heavy  $2p$  emitters.

Prompt  $2p$  decay is a subset of a more general phenomenon of three-body Coulomb decay (TBCD) which exists in mathematical physics (as a formal solution of the  $3 \rightarrow 3$  scattering of charged particles), in atomic physics (as a solution of the  $e \rightarrow 3e$  process), and in molecular physics (as exotic molecules composed from three charged constituents) [3–8]. The theoretical treatment of TBCD is one of the oldest and most complicated problems in physics because of the difficulty associated with the boundary conditions due to the infinite range of the Coulomb force. The exact analytical boundary conditions for this problem are unknown, but different approximations to it have been tried. In nuclear physics, TBCD has not attracted much attention, however the

three-body Coulomb aspect of  $2p$  decay will become increasingly important for heavier prospective  $2p$  emitters [9].

Detailed experimental studies of the correlations have been made for the lightest  $p$ -shell  $2p$  emitter  $^6\text{Be}$  [10, 11] where the Coulomb interactions are minute and their effects are easily masked by the dynamics of the nuclear interactions [12]. The Coulomb effects should be more prominent for the heaviest observed  $2p$  emitters, however these cases are limited by poor statistics; e.g. the latest results for the  $pf$ -shell  $2p$ -emitters  $^{54}\text{Zn}$  [13] and  $^{45}\text{Fe}$  [14] are based on just 7 and 75 events, respectively. Due to these limitations, previous  $2p$  studies dedicated to the long-range treatment of the three-body Coulomb interaction [15], found consistency with the data, but no more.

The present work fills a gap between these previous studies by measuring correlations in the  $2p$  ground-state (g.s.) decay of the  $sd$ -shell nucleus  $^{16}\text{Ne}$  where the Coulombic effects appear to be strong enough to be observable. Known experimentally for several decades [16],  $^{16}\text{Ne}$  has remained poorly investigated with just a few experimental studies [17–20]. However, interest has returned recently with the decay of  $^{16}\text{Ne}$  measured in relativistic neutron-knockout reactions from a  $^{17}\text{Ne}$  beam [21, 22]. We study the same reaction, but at an “intermediate” beam energy and obtain data with better resolution and smaller statistical uncertainty. Combined with state-of-the-art calculations, we find unambiguous evidence for the role of the long-range Coulomb interactions in the measured correlations.

Apart from the Coulomb interactions, predicted cor-

relations show sensitivity to the initial  $2p$  configuration and nuclear final-state interactions that are also evident in  $2n$  decay [23–25]. While there are indications of such sensitivities in  $2p$  data [2], the long-range Coulomb interactions must first be determined accurately before the effects of structure and nuclear final-state interactions can be better probed and properly accounted for.

*Experiment* — A primary beam of  $E/A=170$ -MeV  $^{20}\text{Ne}$ , extracted from the Coupled Cyclotron Facility at the National Superconducting Cyclotron Laboratory at Michigan State University, bombarded a  $^9\text{Be}$  target. The A1900 separator was used to select a secondary  $^{17}\text{Ne}$  beam with a momentum acceptance of  $\pm 1.0\%$ , an intensity of  $\sim 1.5 \times 10^5 \text{ s}^{-1}$ , and a purity of 11% (the largest component was  $^{15}\text{O}$ ). This secondary beam impinged on a 1-mm-thick  $^9\text{Be}$  target with an average of  $E/A=57.6$  MeV in the target's center.

$^{16}\text{Ne}$  decay products were detected in the High Resolution Array (HiRA) [26] in an arrangement of fourteen  $\Delta E - E$  [Si-CsI(Tl)] telescopes subtending zenith angles from  $2^\circ$  to  $13.9^\circ$  [10, 27]. Energy calibrations were achieved using beams of 55 and 75 MeV protons and  $E/A=73$  and 93 MeV  $^{14}\text{O}$ .

*Theoretical model* — The model used in this work is similar to that applied to  $^{16}\text{Ne}$  in [28], but, with improvements concerning basis convergence [29], TBCD [15], and the reaction mechanism [10]. The three-body  $^{14}\text{O}+p+p$  continuum of  $^{16}\text{Ne}$  is described by the wave function (WF)  $\Psi^{(+)}$  with the outgoing asymptotic obtained by solving the inhomogeneous three-body Schrödinger equation,

$$(\hat{H}_3 - E_T)\Psi^{(+)} = \Phi_{\mathbf{q}},$$

with approximate boundary conditions of the three-body Coulomb problem. The three-body part of the model is based on the hyperspherical harmonics method [29]. The differential cross section is expressed via the flux  $j$  induced by the WF  $\Psi^{(+)}$  on the remote surface  $S$ :

$$\frac{d\sigma}{d^3k_{14\text{O}}d^3k_{p_1}d^3k_{p_2}} \sim j = \langle \Psi^{(+)} | \hat{j} | \Psi^{(+)} \rangle \Big|_S. \quad (1)$$

When comparing to the experimental data, the theoretical predictions were used in Monte-Carlo (MC) simulations of the experiment [10, 27] to take into account the apparatus bias and resolution.

The source function  $\Phi_{\mathbf{q}}$  was approximated assuming the sudden removal of a neutron from the  $^{15}\text{O}$  core of  $^{17}\text{Ne}_{\text{g.s.}}$ ,

$$\Phi_{\mathbf{q}} = \int d^3r_n e^{i\mathbf{q}\mathbf{r}_n} \langle \Psi_{14\text{O}} | \Psi_{17\text{Ne}} \rangle, \quad (2)$$

where  $\mathbf{r}_n$  is the radius vector of the removed neutron. The  $^{17}\text{Ne}_{\text{g.s.}}$  WF  $\Psi_{17\text{Ne}}$  was obtained in a three-body model of  $^{15}\text{O}+p+p$  and broadly tested against various observables [30]. Similar ideas had been applied to different reactions populating the three-body continuum of  $^6\text{Be}$  [10–12]. The  $^{14}\text{O}-p$  potential sets were taken from

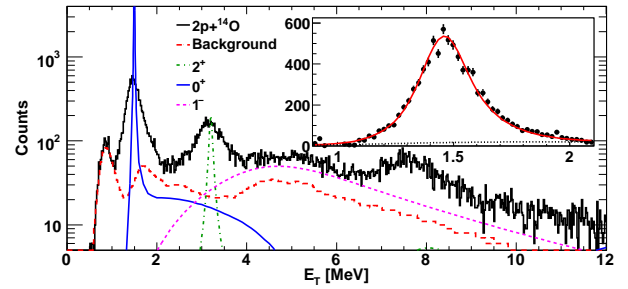


FIG. 1. (Color online) Experimental spectrum of  $^{16}\text{Ne}$  decay energy  $E_T$  reconstructed from detected  $^{14}\text{O}+p+p$  events. The dashed histogram indicates the contamination from  $^{15}\text{O}+p+p$  events. The smooth curves are predictions (without detector resolution) for the indicated  $^{16}\text{Ne}$  states. The inset compares the contamination-subtracted data to the simulation of the g.s. peak for  $\Gamma = 0$ ,  $f_{\text{tar}} = 0.95$ , where the dotted line is the fitted background.

[28] which are consistent with a more recent experiment [31], providing  $1/2^+$  and  $5/2^+$  states at  $E_r = 1.45$  and  $2.8$  MeV, respectively consistent with the experimental properties of these states in both  $^{15}\text{F}$  and  $^{15}\text{C}$ . We used the potential of [32] for the  $p-p$  channel.

The three-body Coulomb treatment in our model consists of two steps. (i) We are able to impose approximate boundary conditions of TBCD on the hypersphere of very large ( $\rho_{\text{max}} \lesssim 4000$  fm) hyperradius by diagonalizing the Coulomb interaction on the finite hyperspherical basis [33]. Within this limitation the procedure is exact, however it breaks down at larger hyperradii as the accessible basis size become insufficient. (ii) Classical trajectories are generated by a MC procedure at the hyperradius  $\rho_{\text{max}}$  and propagated out to distances  $\rho_{\text{ext}} \gg \rho_{\text{max}}$ . The asymptotic momentum distributions are reconstructed from the set of trajectories after the radial convergence is achieved. The accuracy of this approach has been tested in calculations with simplified three-body Hamiltonians allowing exact semi-analytical solutions [15].

*Excitation spectrum* — The spectrum of the total decay energy  $E_T$  constructed from the invariant mass of detected  $^{14}\text{O}+p+p$  events is shown in Fig. 1. Due to a low-energy tail in the response function of the Si  $\Delta E$  detectors, there is leaking of a few  $^{15}\text{O}$  ions into the  $^{14}\text{O}$  gate in the  $\Delta E - E$  spectrum. However, this contamination can be accurately modeled by taking detected  $^{15}\text{O}+p+p$  events and analyzing them as  $^{14}\text{O}+p+p$ . The resulting spectrum (dashed histogram) was normalized to the  $\sim 1$ -MeV peak associated with 2nd-excited state of  $^{17}\text{Ne}$ . All other peaks in the  $^{16}\text{Ne}$  spectrum are associated with  $^{16}\text{Ne}$ , with the g.s. peak at  $E_T = 1.466(20)$  MeV being the dominant feature. This decay energy is consistent with the value of  $1.466(45)$  MeV measured in [19] and almost consistent with, but slightly larger than, other experimental values of  $1.34(8)$  MeV [17],  $1.399(24)$  MeV [18], and  $1.35(8)$  MeV [21],  $1.388(14)$  MeV [22].

The predicted spectra in Fig. 1 provide guidance for possible spin-parity assignments of the other observed structures, suggesting that the previously known peaks [21, 22] at  $E_T = 3.16(2)$  and  $7.60(4)$  MeV are both  $2^+$  excited states. The broad structure at  $E_T \sim 5.0(5)$  MeV is well described as a  $1^-$  “soft” excitation which is not a resonance, but a continuum mode, sensitive to the reaction mechanism [11]. In the mirror  $^{16}\text{C}$  system, there are also  $J = 2^{(\pm)}, 3^{(+)}$ , and  $4^+$  contributions in this energy range, but for neutron-knockout from  $p_{1/2}, p_{3/2}$ , and  $s_{1/2}$  orbitals in  $^{17}\text{Ne}$ , we should only expect *strong* population for  $0^+, 2^+$ , and  $1^-$  configurations. We will concentrate on the g.s. for the remainder of this work ( $1.27 < E_T < 1.72$  MeV) and all subsequent figures will show contamination-subtracted data.

*Three-body energy-angular correlations* — The final state of a three-body decay can be completely described by two parameters [29]: an energy parameter  $\varepsilon$  and an angle  $\theta_k$  between the Jacobi momenta  $\mathbf{k}_x, \mathbf{k}_y$ :

$$\begin{aligned} \varepsilon &= E_x/E_T, & \cos(\theta_k) &= (\mathbf{k}_x \cdot \mathbf{k}_y)/(k_x k_y), \\ \mathbf{k}_x &= \frac{A_2 \mathbf{k}_1 - A_1 \mathbf{k}_2}{A_1 + A_2}, & \mathbf{k}_y &= \frac{A_3(\mathbf{k}_1 + \mathbf{k}_2) - (A_1 + A_2)\mathbf{k}_3}{A_1 + A_2 + A_3}, \\ E_T &= E_x + E_y = k_x^2/2M_x + k_y^2/2M_y, \end{aligned} \quad (3)$$

where  $M_x$  and  $M_y$  are the reduced masses of the  $X$  and  $Y$  subsystems. With the assignment  $k_3 \rightarrow k_{^{14}\text{O}}$ , the correlations are obtained in the “T” Jacobi system where  $\varepsilon$  describes the relative energy  $E_{pp}$  in the  $p$ - $p$  channel. For  $k_3 \rightarrow k_p$ , the correlations are obtained in one of the “Y” Jacobi systems where  $\varepsilon$  describes the relative energy  $E_{\text{core-}p}$  in the  $^{14}\text{O}$ - $p$  channel.

The experimental and predicted (MC simulations) energy-angular distributions, in both Jacobi representations are compared in Fig. 2 and found to be similar. More detailed comparisons will be made with the projected energy distributions.

*The convergence of three-body calculations* is quite slow for some observables [29, 34]. Figure 3 demonstrates the convergence, with increasing  $K_{\text{max}}$  (maximum principle quantum number of the hyperspherical harmonic method) for two observables for which the slowest convergence is expected. This work provides considerable improvement compared to the calculations of [28] which were limited by  $K_{\text{max}} = 20$ .

$^{16}\text{Ne}_{\text{g.s.}}$  *width* — The theoretical difficulty of reproducing the large experimental g.s. widths measured for  $^{12}\text{O}$  and  $^{16}\text{Ne}$  has been pointed out many times in the last 24 years [28, 35–38]. For  $^{12}\text{O}$ , this issue was resolved when a new measurement [39] gave a small upper bound. For  $^{16}\text{Ne}$ , previous measurements of  $\Gamma=200(100)$  keV [17],  $110(40)$  keV [18], and  $82(15)$  keV [22] are large compared to the theoretical predictions, e.g. 0.8 keV in [28].

The experimental resolution is dominated by the effects of multiple scattering and energy loss in the target. Their magnitudes were finely tuned in the MC simulations by reproducing the experimental  $^{15}\text{O}+p+p$  invariant-mass peak associated with the narrow (predicted lifetime of 1.4 fs [40]) 2nd-excited state in  $^{17}\text{Ne}$

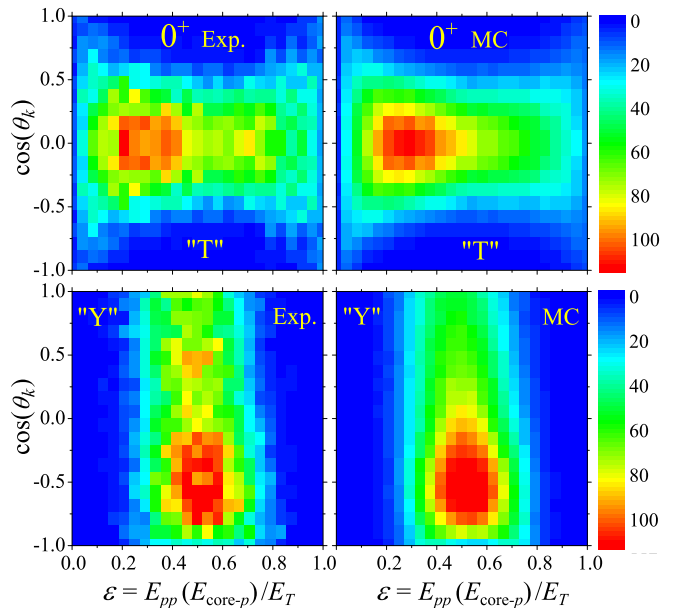


FIG. 2. (Color online) Energy-angular correlations for  $^{16}\text{Ne}_{\text{g.s.}}$ . Experimental and predicted (MC simulations) correlations for Jacobi “T” and “Y” systems are compared.

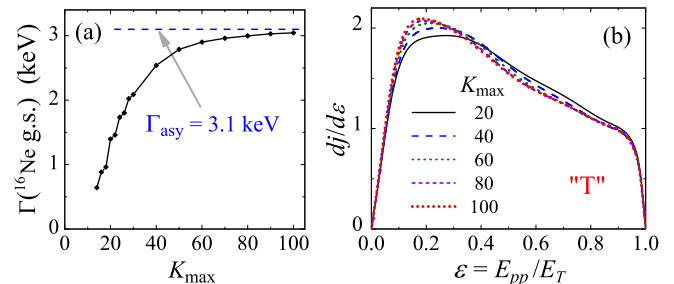


FIG. 3. (Color online) The convergence of the predicted (a) decay width and (b) energy distribution in the “T” system on  $K_{\text{max}}$  (maximum principle quantum number of hyperspherical harmonics method). The asymptotic decay width of  $^{16}\text{Ne}$  assuming exponential  $K_{\text{max}}$  convergence is given in (a) by the dashed line.

by scaling the target thickness from its known value by a factor  $f_{\text{tar}}$ . The best fit is obtained with  $f_{\text{tar}}^{\text{fit}} = 0.95$  with  $3\text{-}\sigma$  limits of 0.91 and 1.00. With  $f_{\text{tar}}^{\text{fit}}$ , we find that the simulated shape of the  $^{16}\text{Ne}_{\text{g.s.}}$  peak for  $\Gamma = 0$  is consistent with the data [Fig. 1 inset]. To obtain a limit for  $\Gamma$ , we used a Breit-Wigner line shape in our simulations and find a  $3\text{-}\sigma$  upper limit of  $\Gamma < 80$  keV with  $f_{\text{tar}} = 0.91$ . This limit is the first experimental result consistent with theoretical predictions of a small width [in the keV range, see, e.g. Fig. 3(a)]. However, our limit is still considerably larger than the predictions, and on the other hand, it is still consistent with two of the previous experiments so even higher resolution measurements are needed to fully resolve this issue.

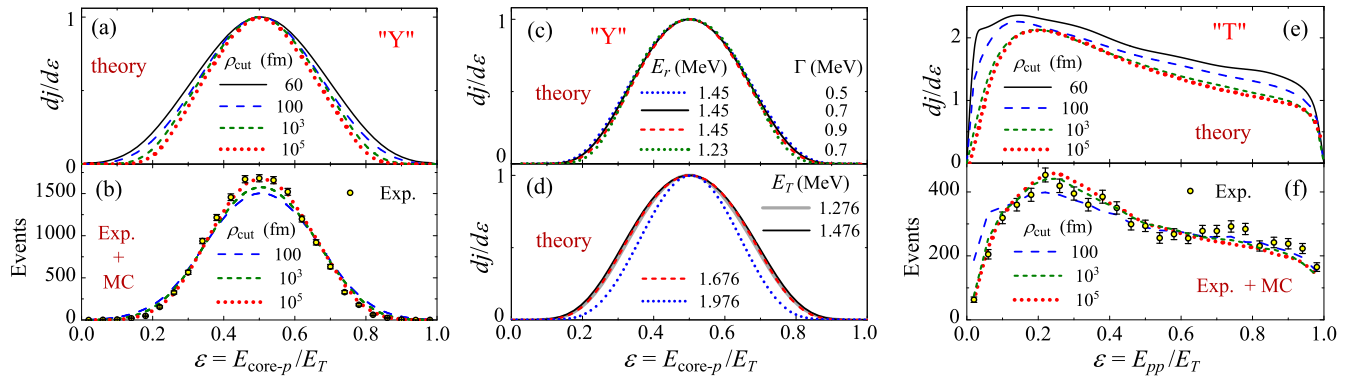


FIG. 4. (Color online) Panels (a)-(d) show energy distributions in the Jacobi “Y” system where (a) gives the sensitivity of the predictions to  $\rho_{\text{cut}}$ , (c) to the  $^{15}\text{F}_{\text{g.s.}}$  properties, and (d) to the decay energy  $E_T$ . Panels (e), (f) show energy distributions in the Jacobi “T” system where (e) gives the sensitivity to  $\rho_{\text{cut}}$ . The theoretical predictions, after the detector bias is included via the MC simulations, are compared to the experimental data in (b) and (f) for the “Y” and “T” systems respectively. The normalization of the theoretical curves is arbitrary, while the MC results are normalized to the integral of the data.

*Evolution of energy distribution between core and proton.* — To investigate the long-range nature of TBCD, we studied the effect of terminating the Coulomb interaction at some hyperradius  $\rho_{\text{cut}}$ . The energy distribution in the “Y” Jacobi system is largely sensitive to just the TBCD and the global properties of the system ( $E_T$ , charges, separation energies) [2]. This makes it most suitable for studying the  $\rho_{\text{cut}}$  dependence [Fig. 4(a)]. Note the arbitrary normalization of the theoretical curves, while the MC results are always normalized to the integral of the data. The comparison with the data in Fig. 4(b) demonstrates consistency with the theoretical calculations only if the considered range of the Coulomb interaction far exceeds  $10^3$  fm ( $\rho_{\text{cut}} = 10^5$  fm guarantees full convergence). This conclusion is only possible due to the high quality of the present data. In contrast in [22], where the experimental width of the g.s. peak is almost twice as large and its integrated yield is  $\sim 3$  times smaller, the corresponding  $\varepsilon$  distribution is broader with a FWHM of 0.41 compared to our value of 0.33. This difference is similar to that obtained over the range of  $\rho_{\text{cut}}$  considered in Fig. 4(a) demonstrating the need for high resolution to isolate these effects.

Our conclusions on TBCD are dependent on the stability of the predicted correlations to the other inputs of the calculations. Figure 4(d) demonstrates the excellent stability of the core- $p$  energy distribution over a broad range ( $\pm 200$  keV) of  $E_T$  centered around  $E_T = 1.476$  MeV. Indeed, in this range we have a maximum in the width for this distribution. This maximum is expected as, below this range, the width must approach zero in the limit of  $E_T \rightarrow 0$  [1] and, above this range, we expect the width to have a minimum at  $E_T \sim 2E_r \sim 2.9$  MeV, where  $E_r$  is the  $^{15}\text{F}_{\text{g.s.}} \rightarrow \text{core} + p$  decay energy. The predictions of such a “narrowing” of the width at  $E_T \sim 2E_r$  [2] were recently proven experimentally [10]. The curve for  $E_T = 1.976$  MeV is also provided in Fig. 4(d) to show that a really large change in energy is required to produce a significant

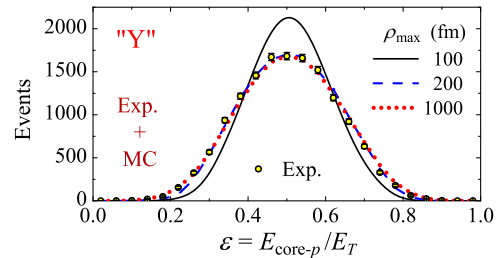


FIG. 5. The core-proton relative-energy distribution (“Y” system) obtained by classical extrapolation started from different  $\rho_{\text{max}}$  values.

modification of the  $\varepsilon$  distribution.

The other important stability issue is with respect to the properties of  $^{15}\text{F}_{\text{g.s.}}$  for which there is *no agreement* on its centroid  $E_r$  and width [41]. Figure 4(c) shows predicted  $\varepsilon$  distributions based on four different  $^{14}\text{O} + p$  interactions which give the indicated  $^{15}\text{F}_{\text{g.s.}}$  properties. Even if we use the data from [42], which differs the most from the other results ( $E_r \sim 1.23$  MeV instead of  $E_r \sim 1.4 - 1.5$  MeV), no drastic effect is seen.

*The evolution of energy distribution between two protons with  $\rho_{\text{cut}}$*  is shown in Fig. 4(e). This distribution has greater sensitivity to the initial  $2p$  configuration of the decaying system [2]. In addition, the spin-singlet interaction in the  $p$ - $p$  channel provides the virtual state (“diproton”) which also can affect the long-range behavior of the correlations (see [23–25] for the corresponding effects in  $2n$  decay). The theoretical prediction for  $\rho_{\text{cut}} = 10^5$  fm in Fig. 4(f) reproduces experimental data quite well, however, the sensitivity to  $\rho_{\text{cut}}$  is diminished compared to the core- $p$  energy distribution.

*Limits on classical motion* — In our model the very long distances are achieved by classical extrapolation. This approximation has been studied using calculations

with simplified Hamiltonians where it was demonstrated that the classical extrapolation provides stable results if the starting distance  $\rho_{\max}$  exceeds some hundreds of fermis for  $E_T \sim 1$  MeV [15]. (e.g.  $\sim 300$  fm for  $^{19}\text{Mg}_{g.s.}$  decay where  $E_T = 0.75$  MeV). At such distances, the ratio of the Coulomb potential to the kinetic energy of fragments is of the order  $10^{-2}$ – $10^{-3}$ . Figure 5 shows that for  $^{16}\text{Ne}_{g.s.}$ , the predictions are consistent with the data only if the conversion from quantum to classical dynamics is made at or above 200 fm.

*Conclusions* — The continuum of  $^{16}\text{Ne}$  has been studied both experimentally and theoretically with emphasis on the ground state which decays by prompt two-proton emission. The measured decay correlations in this work were found to require a theoretical treatment in which the three-body Coulomb interaction is considered out to distances far beyond  $10^3$  fm. Our theoretical treatment is now validated for use in interpreting the results of future studies of heavier two-proton decay with particular emphasis on extracting nuclear-structure information from

correlation observables.

We extract a limit of  $\Gamma < 80$  keV for the intrinsic decay width of the ground state, and while this is not inconsistent with some of the previous measurements, it is the first measurement consistent with the theoretical predictions. All conclusions of this work were only possible due to the high statistics and fidelity of the present measurements.

*Acknowledgments* — This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Award numbers DE-FG02-87ER-40316 and DE-FG02-04ER41320 and the National Science Foundation under grants PHY-1102511 and PHY-9977707. I.A.E. is supported by the Helmholtz Association under grant agreement IK-RU-002 via FAIR-Russia Research Center and L.V.G. by the RFBR 14-02-00090 and Russian Ministry of Industry and Science NSH-932.2014.2 grants. K.W.B. is supported by the National Science Foundation Graduate Research Fellowship under grant No. DGE-1143954.

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